Modes in Multimode Fibers

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December 10, 2015

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Chapter 1

Step-index Multimode Fibers

1.1 Introduction

This summary is largely inspired by chapter 3 of *Fundamentals of Optical Waveguides* by K. Okamoto [1].

We consider a fiber of core radius a of refractive index n_1 surrounded by a cladding of index n_0. We assume that we satisfy in the weakly guided approximation, *i.e*:

$$\Delta = \frac{n_1 - n_0}{n_1} \ll 1 \tag{1.1}$$

In practice, this approximation is quite accurate as we have $\Delta \leq 0.1\%$ in standard fibers.

1.2 Dispersion relation of Linearly Polarized (LP) modes

In the weakly guided approximation, we can define Linearly Polarized (LP) modes obtained by combination of the TE, TM and hybrid modes supported by the fiber that has the same propagation constant. They satisfy the general dispersion relation:

$$\frac{J_m(u)}{uJ_{m-1}(u)} = \frac{K_w(u)}{wK_{m-1}(w)} \tag{1.2}$$

with J_m (K_m) is the Bessel function of the first (second) kind of order m and

$$u = a\sqrt{k^2 n_1^2 - \beta^2} \tag{1.3}$$

$$w = a\sqrt{\beta^2 - k^2 n_0^2} \tag{1.4}$$

$k = 2\pi/\lambda$ and β is the propagation constant.

The modes are indexed by two integers $m \geq 0$ and $l \geq 1$. For a given m, the integer l numbers the solutions of Eq.1.4. For $m = 0$, the combination (m, l) is two fold degenerate, for $m \geq 1$, the combination (m, l) is four fold degenerate.

1.3 Spatial profile of the LP modes

1.3.1 TE, TM and hybrid modes

By solving the wave equation, one finds the different solutions corresponding to Transverse Electric (TE) modes, Transverse Magnetic (TE) modes and hybrid modes (called HE and EH).

TE modes

For $r \leq a$:

$$E_\theta = -j\omega\mu_0 \frac{a}{u} A J_1\left(\frac{u}{a}r\right) \tag{1.5}$$
$$H_r = j\beta_l \frac{a}{u} A J_1\left(\frac{u}{a}r\right) \tag{1.6}$$
$$H_z = A J_0\left(\frac{u}{a}r\right) \tag{1.7}$$
$$E_r = E_z = H_\theta = 0 \tag{1.8}$$

For $r > a$:

$$E_\theta = j\omega\mu_0 \frac{a}{w} \frac{J_0(u)}{K_0(w)} A K_1\left(\frac{w}{a}r\right) \tag{1.9}$$
$$H_r = -j\beta \frac{a}{w} A \frac{J_0(u)}{K_0(w)} A K_1\left(\frac{w}{a}r\right) \tag{1.10}$$
$$H_z = A \frac{J_0(u)}{K_0(w)} A K_0\left(\frac{w}{a}r\right) \tag{1.11}$$
$$E_r = E_z = H_\theta = 0 \tag{1.12}$$

A is a normalization constant.

TM modes

For $r \leq a$:

$$E_r = j\beta_l \frac{a}{u} A J_1\left(\frac{u}{a}r\right) \tag{1.13}$$
$$E_z = A J_0\left(\frac{u}{a}r\right) \tag{1.14}$$
$$H_\theta = j\omega\epsilon_0 n_1^2 A J_1\left(\frac{u}{a}r\right) \tag{1.15}$$
$$E_\theta = H_r = H_z = 0 \tag{1.16}$$

For $r > a$:

$$
\begin{aligned}
E_r &= -j\beta\frac{a}{w}A\frac{J_0(u)}{K_0(w)}K_1\left(\frac{w}{a}r\right) && (1.17)\\
E_z &= A\frac{J_0(u)}{K_0(w)}K_0\left(\frac{w_l}{a}r\right) && (1.18)\\
H_\theta &= -j\omega\epsilon_0 n_0^2\frac{a}{w}A\frac{J_0(u)}{K_0(w)}K_1\left(\frac{w}{a}r\right) && (1.19)\\
E_\theta &= H_r = H_z = 0 && (1.20)
\end{aligned}
$$

Hybrid modes (HE and EH modes)

For $r \leq a$:

$$
\begin{aligned}
E_r &= -jA\beta\frac{a}{u}\left[\frac{1-s}{2}J_{n-1}\left(\frac{u}{a}r\right) - \frac{1+s}{2}J_{n+1}\left(\frac{u}{a}r\right)\right]\cos(n\theta+\psi) && (1.21)\\
E_\theta &= jA\beta\frac{a}{u}\left[\frac{1-s}{2}J_{n-1}\left(\frac{u}{a}r\right) + \frac{1+s}{2}J_{n+1}\left(\frac{u}{a}r\right)\right]\sin(n\theta+\psi) && (1.22)\\
E_z &= AJ_n\left(\frac{u}{a}r\right)\cos(n\theta+\psi) && (1.23)\\
H_r &= -jA\omega\epsilon_0 n_1^2\frac{a}{u}\left[\frac{1-s}{2}J_{n-1}\left(\frac{u}{a}r\right) + \frac{1+s}{2}J_{n+1}\left(\frac{u}{a}r\right)\right]\sin(n\theta+\psi) && (1.24)\\
H_\theta &= -jA\omega\epsilon_0 n_1^2\frac{a}{u}\left[\frac{1-s}{2}J_{n-1}\left(\frac{u}{a}r\right) - \frac{1+s}{2}J_{n+1}\left(\frac{u}{a}r\right)\right]\cos(n\theta+\psi) && (1.25)\\
H_z &= -A\frac{\beta}{\omega\mu_0}sJ_n\left(\frac{u}{a}r\right)\sin(n\theta+\psi) && (1.26)
\end{aligned}
$$

For $r > a$:

$$
\begin{aligned}
E_r &= -jA\beta\frac{aJ_n(u)}{wK_n(w)}\left[\frac{1-s}{2}K_{n-1}\left(\frac{w}{a}r\right) + \frac{1+s}{2}K_{n+1}\left(\frac{w}{a}r\right)\right]\cos(n\theta+\psi) && (1.27)\\
E_\theta &= jA\beta\frac{aJ_n(u)}{wK_n(w)}\left[\frac{1-s}{2}K_{n-1}\left(\frac{w}{a}r\right) - \frac{1+s}{2}K_{n+1}\left(\frac{w}{a}r\right)\right]\sin(n\theta+\psi) && (1.28)\\
E_z &= A\frac{J_n(u)}{K_n(w)}K_n\left(\frac{w}{a}r\right)\cos(n\theta+\psi) && (1.29)\\
H_r &= -jA\omega\epsilon_0 n_0^2\frac{aJ_n(u)}{wK_n(w)}\left[\frac{1-s}{2}K_{n-1}\left(\frac{w}{a}r\right) - \frac{1+s}{2}K_{n+1}\left(\frac{w}{a}r\right)\right]\sin(n\theta+\psi) && (1.30)\\
H_\theta &= -jA\omega\epsilon_0 n_0^2\frac{aJ_n(u)}{wK_n(w)}\left[\frac{1-s}{2}K_{n-1}\left(\frac{w}{a}r\right) + \frac{1+s}{2}K_{n+1}\left(\frac{w}{a}r\right)\right]\cos(n\theta+\psi) && (1.31)\\
H_z &= -A\frac{\beta}{\omega\mu_0}s\frac{J_n(u)}{K_n(w)}K_n\left(\frac{w}{a}r\right)\sin(n\theta+\psi) && (1.32)
\end{aligned}
$$

with $s = \pm 1$. By convention, we call EH modes for $s = 1$ and HE modes for $s = -1$.

1.3.2 Relation between LP modes and TE, TM and hybrid modes

LP$_{0l}$ modes

For $m = 0$, the LP modes correspond to the hybrid modes HE$_{11}$ with $\psi = 0$ and $\psi = \pi/2$. We then have two degenerate modes corresponding to the same mode profile but with two orthogonal linear polarizations.

$$LP_{0l} = HE_{1l}(\psi = 0) \quad (1.33)$$

$$= HE_{1l}(\psi = \pi/2) \quad (1.34)$$

where $\beta = \beta_{ml}$ obtained finding the solutions of the dispersion relation.

LP$_{1l}$ modes

The linearly polarized modes for $m = 1$ are obtained by the superpositions of the TE mode and the HE$_{2l}$ mode for $\psi = \pi/2$ and by the superpositions of the TM mode and the HE$_{2l}$ mode for $\psi = 0$.

$$LP_{1l} = TE + HE_{2l}(\psi = \pi/2) \quad (1.35)$$

$$= TE - HE_{2l}(\psi = \pi/2) \quad (1.36)$$

$$= TM + HE_{2l}(\psi = 0) \quad (1.37)$$

$$= TM - HE_{2l}(\psi = 0) \quad (1.38)$$

where $\beta = \beta_{ml}$ obtained finding the solutions of the dispersion relation.

LP$_{ml}$ modes, $m > 1$

The linearly polarized modes for $m > 1$ are obtained by the superposition of the hybrid modes HE$_{m-1l}$ and EH$_{m+1l}$ for $\psi = 0$ and $\psi = \pi/2$.

$$LP_{ml} = HE_{m+1l}(\psi = 0) + EH_{m-1l}(\psi = 0) \quad (1.39)$$

$$= HE_{m+1l}(\psi = 0) - EH_{m-1l}(\psi = 0) \quad (1.40)$$

$$= HE_{m+1l}(\psi = \pi/2) + EH_{m-1l}(\psi = \pi/2) \quad (1.41)$$

$$= HE_{m+1l}(\psi = \pi/2) - EH_{m-1l}(\psi = \pi/2) \quad (1.42)$$

where $\beta = \beta_{ml}$ obtained finding the solutions of the dispersion relation.

1.3.3 General expression of the LP modes

The previous expressions car be simplified using trigonometric relations to obtain a general expression for the LP modes [2, 3]. For any fixed orthonormal coordinate system [X,Y,Z] with Z the propagation axis, the electric field of modes can be expressed :

LP_{ml}

For $r \leq a$:

$$E_{x,y} = -jA\beta\frac{a}{u}J_{l-1}\left(\frac{u}{a}r\right)\cos\left(m\theta+\psi\right) \tag{1.43}$$

$$E_{y,x} = 0 \tag{1.44}$$

$$E_z = AJ_l\left(\frac{u}{a}r\right)\cos\left(m\theta+\psi\right) \tag{1.45}$$

For $r > a$:

$$E_{x,y} = -jA\beta\frac{a}{u}J_{l-1}\left(\frac{u}{a}r\right)\cos\left(m\theta+\psi\right) \tag{1.46}$$

$$E_{y,x} = 0 \tag{1.47}$$

$$E_z = A\frac{J_l\left(u\right)}{K_l\left(w\right)}K_l\left(\frac{w}{a}r\right)\cos\left(m\theta+\psi\right) \tag{1.48}$$

with $\psi = 0$ for $m = 0$ and $\psi = 0$ or $\psi = \pi$ for $m > 0$ (degenerate modes).

1.3.4 Visual aspect of the LM modes

We present if figure 1.1 the spatial profile of the first four LP modes. If we define two orthogonal directions x and y orhtogonal to the optical fiber axis z, for each of these figures, there exists two modes; one for which $E_y = 0$ and E_x has the spatial profile represented in figure 1.1 and a mode where $E_x = 0$ and E_y has the spatial profile of the LP mode calculated.

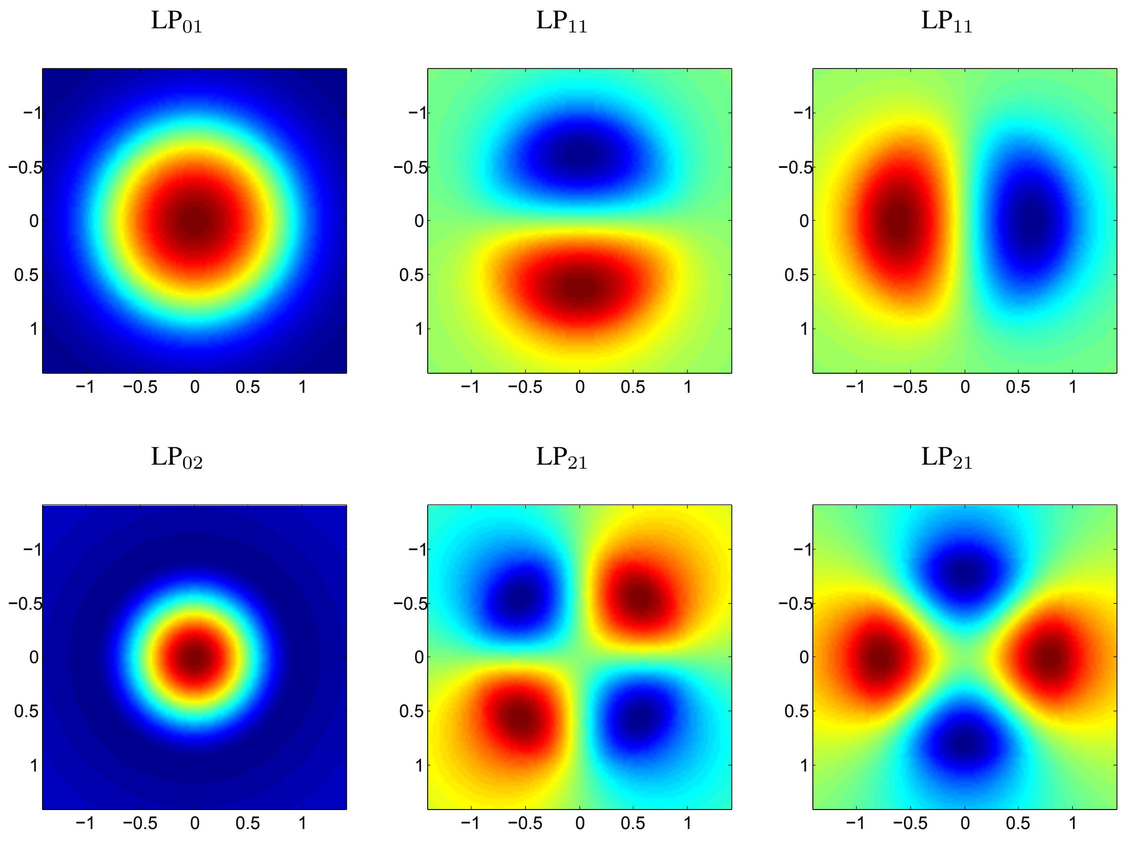

Figure 1.1: Spatial profile of the first LP modes. Distance units are normalized by the radius a.

Bibliography

[1] Katsunari Okamoto. Fundamentals of optical waveguides. Academic press, 2010.

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